

YBMA News

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The Newsletter of the Yorkshire Branch of
the Association for
Mathematics in
Education



The first anniversary of the formal merging of the five maths teaching oriented associations is fast approaching. Before the merger each association had its own focus and priorities and all were determined that their own character should not be lost in the process. The merger could only take place after it was realised that a unified, rigidly structured entity would not be agreed upon prior to holding a vote to join up. Many of the activities of the separate associations would be carried over and continue unchanged for some time. The structure of AMiE would emerge in response to the needs and demands of its members.

We are still only part of the way along this process. An AMiE eNewsletter first appeared in March and monthly since. We had a Members' Day in April, an online FE conference in May and an online AGM in June. At the AGM, in his [report](#) as Chair of the Transitional Board, Andy Kemp encouraged members to get involved in the Member Interest Communities (MICs). He emphasised that these should not be seen as optional, peripheral activities but very much central to shaping the development of AMiE. Leads for the MICs have been appointed and their work has begun. A link in the most recent eNewsletter allows AMiE members to [register their interest](#) in any or all MICs. Please consider taking part, if only to remain informed about what is happening.

Our [Yorkshire Branch page](#) is still accessible to all on what is essentially the old MA website. We hope that this will continue to be updated, at least until the promised new AMiE website is up and running.

Spring & Summer Programme 2027

Full details will be available shortly.

Yorkshire Branch Officers 2026-27

President: Tom Roper (ropertom@outlook.com)

Secretary & Newsletter: Bill Bardelang (rgb43@gmx.com)

Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

Autumn Meeting

Wednesday, 14 October 2026
7pm for 7.30pm

MALL 1, School of Mathematics
University of Leeds

Dr James Cranch

University of Sheffield

Which Real Numbers are nice?

Mathematicians need nice numbers to work with. For example, the integers and rationals are certainly nice, but unfortunately there's not enough of them to keep us happy. We'll talk about some more general classes of nice real numbers. I'll end up talking about some classes of numbers that some experts conjecture the mathematicians of the future will be able to work with.

Refreshments will be provided

Christmas Quiz

Wednesday, 2 December 2026
7pm for 7.30pm

MALL 1, School of Mathematics
University of Leeds

Seasonal Food and Drink!
Prizes!

Come and enjoy yourself!
Bring a friend or colleague!

In recent years the role of question master has been shared by willing participants. If you are able to contribute in this way, please bring along a round of questions. These can be in any format you choose and the suggested time allowance is 5 to 10 minutes. Some connection with Christmas and/or Mathematics is desirable, but not essential.

The Sum of two Cubes

Whether a natural number N can be expressed as the sum of two squares depends on its prime factorisation. N is the sum of two squares if and only if every prime factor of the type $4n+3$ occurs as an even power. We give some examples:

$$45 = 3^2 \times 5 = 6^2 + 3^2, \quad 52 = 2^2 \times 13 = 6^2 + 4^2, \quad \text{but not } 60 = 2^2 \times 3 \times 5 \text{ nor } 63 = 3^2 \times 7.$$

Moreover, if N cannot be expressed as the sum of two integer squares, then it cannot be expressed as the sum of two rational squares.

Turning our attention to the sum of two cubes, the following example shows that the situation is very different. Suppose $7 = a^3 + b^3$. We can find $7 = 2^3 + (-1)^3$, but there is no solution using only positive integers.

Now suppose that $a^3 + b^3 = N c^3$ for some non-zero integers a, b, c with $c > 1$. If so, then there is a rational solution $N = (a/c)^3 + (b/c)^3$. We will search for some solutions using small positive values a, b .

$$2^3 + 1^3 = 9 = 3^2$$

$$3^3 + 1^3 = 28 = 2^2 \cdot 7$$

$$3^3 + 2^3 = 35 = 5 \cdot 7$$

$$4^3 + 1^3 = 65 = 5 \cdot 13$$

$$4^3 + 3^3 = 91 = 7 \cdot 13$$

$$5^3 + 1^3 = 126 = 2 \cdot 3^2 \cdot 7$$

$$5^3 + 2^3 = 133 = 7 \cdot 19$$

$$5^3 + 3^3 = 152 = 2^3 \cdot 19$$

$$5^3 + 4^3 = 189 = 3^3 \cdot 7$$

With very little effort we have found $19 = (5/2)^3 + (3/2)^3$ and $7 = (5/3)^3 + (4/3)^3$. Note that we now have two solutions for $N = 7$. The integer solution one might regard as a difference rather than a sum, but the rational solution is a 'true' sum.

An ingenious algebraic identity relates our second solution to our first. We will construct this identity in easy stages.

$$x(x - y)^3 = x^4 - 3x^3y + 3x^2y^2 - xy^3$$

$$y(x - y)^3 = x^3y - 3x^2y^2 + 3xy^3 - y^4$$

Hence
$$(x + y)(x - y)^3 = x^4 - 2x^3y + 2xy^3 - y^4$$

Also
$$x(x + 2y)^3 = x^4 + 6x^3y + 12x^2y^2 + 8xy^3$$

$$y(2x + y)^3 = 8x^3y + 12x^2y^2 + 6xy^3 + y^4$$

Hence
$$x(x + 2y)^3 - y(2x + y)^3 = x^4 - 2x^3y + 2xy^3 - y^4$$

Thus
$$(x + y)(x - y)^3 = x(x + 2y)^3 - y(2x + y)^3$$

Let $x = a^3, y = b^3$. Then
$$(a^3 + b^3)(a^3 - b^3)^3 = a^3(a^3 + 2b^3)^3 - b^3(2a^3 + b^3)^3$$

We leave it to the reader to verify that for $a = 2, b = -1$ this identity leads to the rational solution we found earlier. Using the new rational values in turn we can obtain a further rational solution, but this includes a negative value. Can you find a further all-positive rational solution?

Whenever we can find just a single pair of integer or rational cubes whose sum or difference is equal to N , then we can find more. For many values of N there are no solutions, e.g. for $N = 3, 4$ and 5 this was proven by Euler and Legendre.

We challenge the reader to find additional integer solutions of $a^3 + b^3 = N c^3$ for $c = 2$ and $c = 3$. These will give us values of N expressible as sums of two cubes of halves and thirds respectively.

Tetraflexagons - Solutions

In the May Newsletter we invited readers to construct three distinct tetra-tetraflexagons.

(i) This most obvious extension of the tri-tetraflexagon can be made from the single piece shown. The labels to go on the back of each square are given by the subscripts. Start folding together adjacent 4s, then 3s. This leaves all 1s on the front and all 2s on the back. Join the loose ends to close the loop. It flexes just like the tri-tetraflexagon but with three configurations instead of two.

1_2	3_2	4_1	4_3
4_3	4_1	3_2	1_2

$$[1,2] \xrightarrow{\text{flex}} [3,1] \xrightarrow{\text{flex}} [4,3]$$

(ii) Shown below is the single piece required for this variation. Surprisingly, we don't reach a dead-end when flexing, instead cycling through four distinct configurations. After each flex the flexagon needs to be rotated in its own plane by a quarter turn before the next flex.

			1_3
	1_2	4_2	4_3
	1_3		
1_2	4_2	4_3	

$$[1,2] \xrightarrow{\text{turn \& flex}} [4,1]$$

↑
turn
&
flex

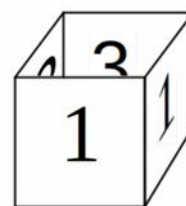
↓
turn
&
flex

$$[2,3] \xleftarrow{\text{turn \& flex}} [3,4]$$

(iii) The single piece alternative is shown below. Again four configurations arise and in the same cyclic sequence as for (ii). However, a "tube flex" is needed for two of the four transitions.

To go from $[1,2]$ to $[4,1]$, start as usual by folding to hide the 2s, but then open out into a tube with 1s on the outside and 3s on the inside. Close the tube along the other diagonal hiding the 3s. Now we can open the flexagon out to reveal the 4s.

The transition from $[3,4]$ to $[2,3]$ is another "tube flex".



			4_1	4_3
		1_2	3_2	
	4_1	4_3		
1_2	3_2			

$$[1,2] \xrightarrow{\text{tube flex}} [4,1]$$

↑
turn
&
flex

↓
turn
&
flex

$$[2,3] \xleftarrow{\text{tube flex}} [3,4]$$